

## Growth probability distribution in percolation

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1991 J. Phys. A: Math. Gen. 24 1593

(<http://iopscience.iop.org/0305-4470/24/7/030>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 129.252.86.83

The article was downloaded on 01/06/2010 at 14:11

Please note that [terms and conditions apply](#).

## Growth probability distribution in percolation

S S Manna† and Naeem Jan

Department of Physics, St Francis Xavier University, Antigonish, Nova Scotia, Canada  
B2G 1C0

Received 29 October 1990

**Abstract.** We study the growth probability distribution for a single cluster growth process for percolation at the percolation threshold. We find that this distribution is indistinguishable from the chemical path distribution on the percolation cluster as studied by Havlin *et al.* We also study the density profile of the percolation cluster. Using the multiscaling idea of Coniglio and Zannetti we calculate the fractal dimension of the percolation clusters from the density profile and report  $d_f = 1.895 \pm 0.005$  for two dimensions and  $2.50 \pm 0.01$  for three dimensions. However we do not find any evidence of multifractal or multiscaling behaviour.

What is the probability that a new particle will be a member of a growing cluster at a distance  $r$  from the centre when the cluster has grown up to a certain size? This question is important when one tries to understand the structural properties of the cluster as a whole and the surface structure in particular at the infinite size limit.

For a growing cluster, growth always takes place at the surface of the cluster. The addition of a new particle to the surface is governed by definite rules and variation of these rules leads to different growth processes. As an example, let us consider the diffusion limited aggregation process (DLA). In this case a cluster grows from a single seed, particles come from large distances in a diffusion process, and whenever one particle is close to the surface, it sticks to the cluster and thus growth takes place. Plischke and Racz [1] showed that surface of any growing cluster is divided into two distinct regions. One is the active zone, usually the outer portion of the cluster, which collects most of the new particles. The other is the passive zone which is the frozen core region around the centre, where growth has almost stopped. One can define a perimeter of the cluster by locating those unoccupied sites which are the neighbouring positions of occupied sites of the cluster and where growth can take place in principle. Meakin *et al* [2] observed that in the DLA process, different perimeter sites have different growth probabilities and they defined a growth site probability distribution (GSPD) to characterize the perimeter sites according to the growth probability. Plischke and Racz [1] studies the probability distribution  $P(r, N)$  of the  $(N + 1)$ th particle to be a member of the cluster at a distance  $r$  from the origin when the cluster has grown up to mass  $N$ . They obtained a Gaussian distribution for this probability, characterized by the average radius of the growing zone  $\langle r \rangle$ , the width of the interface  $\xi$  and the radius of gyration  $R$  as function of the particle number  $N$ .

† Present address: HLRZ c/o KFA Julich GmbH, Postfach 1913, D-5170 Julich 1, Federal Republic of Germany.

We study here these questions for percolation at the percolation threshold. We study cluster growth from a single seed. The algorithm was first proposed by Leath [3] for all clusters and later modified by Alexandrowicz [4] for a single cluster. Here the central seed site is occupied with probability 1. In time step one, for each of the nearest neighbour sites of the central seed, a random number is compared with the percolation threshold  $p_c$ . Sites with random numbers less than the threshold are occupied and the other sites with random numbers greater than  $p_c$  are blocked. These newly occupied sites are the potential 'parent' sites for growth at the next time step; their unoccupied and unblocked neighbouring sites are compared with  $p_c$  to obtain the growth sites for the next time step, the process continues. The chemical distance,  $l$  between two points on a cluster is the length of the shortest path connecting the points. Therefore the time, at which a site has become a member of the cluster, is exactly equal to the chemical distance when measured from the origin. It may happen that at a certain time there are no further growth sites at all and therefore growth is totally blocked. In this situation we discard this cluster and consider only those clusters which have grown up to a certain chemical distance  $l_{\max}$ . The most important difference between the percolation and DLA growth processes is that in the case of percolation, at any time all perimeter sites are equally probable to be a part of the cluster at the next time step, whereas in DLA some sites are more probable than others. We grow percolation clusters at the percolation threshold using the recent accurate estimate of the threshold  $p_c = 0.592746$  for the square lattice [5] and  $p_c = 0.311605$  for the simple cubic lattice [6]. We consider only those clusters which have grown up to a chemical distance  $l_{\max}$  and we observed success rates of 0.588 and 0.325 for  $l_{\max} = 1024$  and 100 in two and three dimensions.

We first study how the radius  $\langle r \rangle$  and the width  $\xi$  of the interface of the growing region scales with chemical distance  $l$

$$\langle r \rangle \sim l^{\nu_1} \quad (1)$$

$$\xi = \sqrt{\langle r^2 \rangle - \langle r \rangle^2} \sim l^{\nu_2} \quad (2)$$

and the average radius of gyration  $R$ , defined as

$$R = \sqrt{\langle r^2 \rangle} \sim l^{\nu_3} \quad (3)$$

We calculate these quantities for the square lattice. For each cluster we keep track of the history of the growth and calculate the quantities for  $l = 16$  to  $l_{\max}$  at the interval of 16 for an ensemble of 100000 clusters. For each value of  $l$  we consider only those particles added to the cluster at that chemical distance. We plot these three quantities for different chemical distances  $l$  on a double logarithmic scale (see figure 1). The curves are fairly straight and our estimate for the slopes are  $\nu_1 = 0.8782$ ,  $\nu_2 = 0.9062$  and  $\nu_3 = 0.8758$ . To see whether  $\nu_1$  and  $\nu_2$  are the same or not we plot  $\langle r \rangle / \xi$  with  $\log l$  in figure 1(a). The curve asymptotes to a constant value for large  $l$ . We infer that  $\langle r \rangle$  and  $\xi$  are characterized by the same critical exponent  $\nu$ .

Next we study the probability distribution of a new particle to be a member of the growing cluster at a distance  $r$  from the centre when the cluster has grown up to a certain chemical distance  $l$ . We again consider clusters grown up to chemical distance  $l_{\max}$  and only for the successful clusters we look back and consider only those particles which became a member of the growing cluster at different chemical distances  $l = 64, 128, 256, 512$  and  $1024$  and average over 100 000 clusters. For each particle we calculate the distance from the origin and truncate it to the nearest integer and update the corresponding location in a storing array. We plot this probability  $P(r, R)$  with  $r$  for

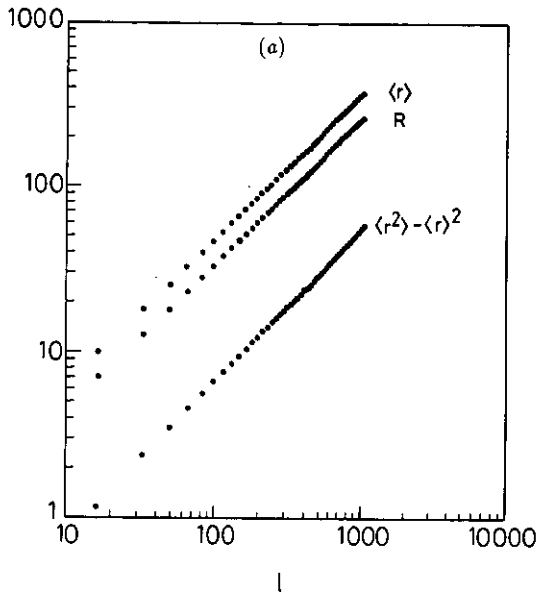


Figure 1. Double logarithmic plot of average radius  $\langle r \rangle$ , the width of the interface  $\xi = \langle r^2 \rangle - \langle r \rangle^2$  of the growing region and the average radius  $R$  of gyration of the whole cluster with chemical distance  $l$ . The slopes of all three curves are almost identical.

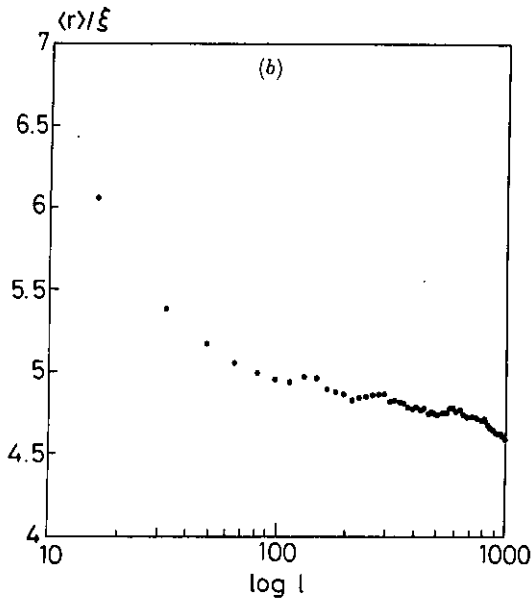


Figure 1(a). The ratio  $\langle r \rangle / \xi$  is plotted against  $\log l$ . This ratio is tending towards a constant for large values of  $l$ .

different radius of gyrations corresponding to different chemical distances  $l$  (see figure 2). We see that on increasing chemical distance the distribution becomes more and more flat and consequently the height decreases. To get a scaled distribution we contract the abscissa by  $R$  and expand ordinate by  $R$ . On a plot of  $P(r, R) \times R$  vs  $(r/R)$  we see a very nice data collapse of data for all five different chemical distances (see figure 3).

Havlin *et al* [7] studied the probability distribution of the end to end distance of a self-avoiding walk (SAW), which is defined by the shortest path between the two points (i.e., the chemical distance), on the percolation cluster. Here, the scaled ( $x = r/\langle r \rangle$ ) distribution follows the following form:

$$P(x) = Ax^{-g} \exp(-ax^\delta). \quad (4)$$

Comparing this distribution with the distribution of SAWs [8] they recognized the exponent  $\delta = 1/(1 - \nu)$ . In our case, the growth probability distribution is the distribution of chemical paths which start from the centre. We get a very nice fit to the form (4) with the values of the constants  $A = 1.15$ ,  $g = 2.5$ ,  $a = 2.7$ ,  $\phi = 9.8$  and we had to contract the  $x$  axis by a factor  $f = 0.485$  because we used  $x = r/R$  and not  $r/\langle r \rangle$ . These values are in excellent agreement with those reported in reference [7].

Similar growth probability distribution on the simple cubic lattice at the percolation threshold is studied for chemical distances  $l = 20, 40, 60, 80$  and  $100$  (see figure 4). After a similar scaling as before with the radius of gyration  $R$  we plot  $P(r, R)$  and  $r/R$  (see figure 5). Here also we see very well data collapse. Similar fit with the functional form in (4) is tried and the constants obtained are  $A = 1.0$ ,  $g = 2.7$ ,  $a = 95.0$  and  $\delta = 3.4$  where  $x$  axis is contracted by a factor  $f = 0.175$ . Unlike the case of the square lattice, the asymmetry is almost absent in the three dimensional distribution. This is reflected in the very high value of the constant  $a$  in the fitting distribution.

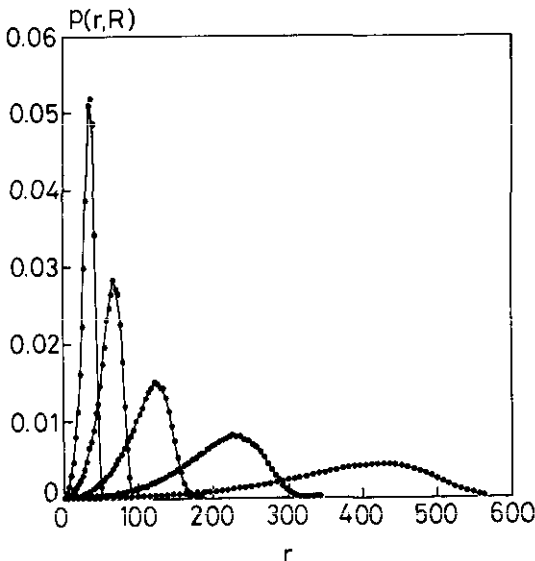
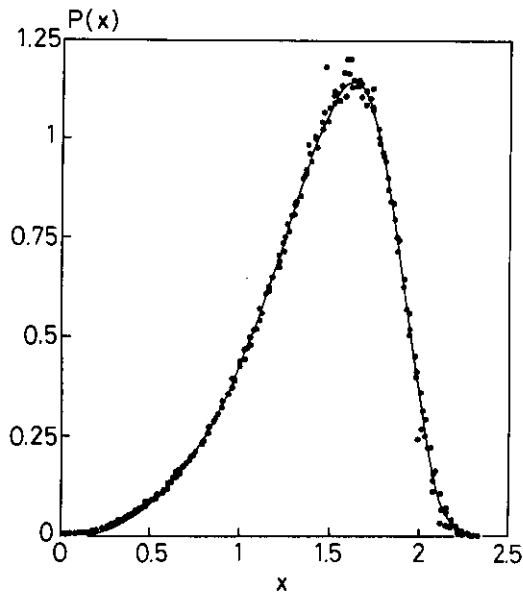
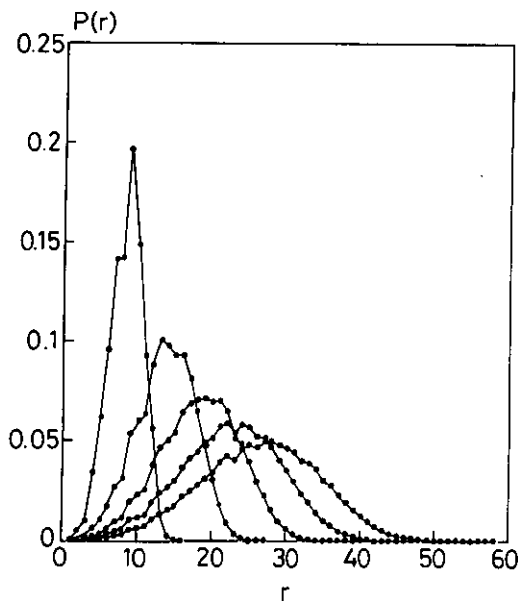


Figure 2. The probability distribution  $P(r, R)$  of a new growth site to be a member of the growing cluster at a chemical distance  $l$  (corresponding to the radius of gyration  $R$ ) is plotted for different chemical distances  $l = 64, 128, 256, 512$  and  $1024$  from left to right.



**Figure 3.** Collapse of the data used in figure 2 for the growth site probability distribution  $P(r, R)$ . The abscissa is now  $r/R$  whereas the ordinate is  $P(r, R) \times R$ .



**Figure 4.** Similar plot of the growth site probability distribution as in figure 2 but for simple cubic lattice. Curves from left to the right are for the chemical distances  $l = 20, 40, 60, 80$  and  $100$ .

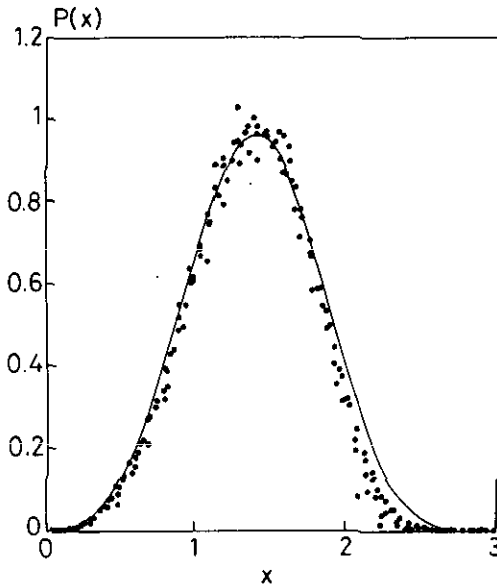


Figure 5. Similar data collapse as in figure 3 but for simple cubic lattice.

Next we calculate the density profile  $g(r, R)$  in a percolation cluster grown up to chemical distance  $l$ , as a function of  $r$ . For that we again generate clusters up to a maximum size  $l_{\max} = 1024$  and then consider clusters at five different stages of growth with  $l$  values of 64, 128, 256, 512 and 1024. For each  $l$  values we superpose  $N = 100\,000$  clusters over one another. If a site is occupied  $n$  times out of  $N$  configurations, then the average density at that site is  $n/N$ . We integrate this density over all sites on an annular ring of width  $dr$  between  $r$  and  $r + dr$  and divide this value by the number of sites in the ring to get  $g(r, R)$ . Average density calculated in this way actually represents the site occupation correlation between the central site and any other site at a distance  $r$ . We plot  $g(r, R)$  vs  $r$  for different  $l$  values (see figure 6). Here  $r=0$  corresponds to the origin for which the density is always 1 and then it decreases with increasing  $r$ . Similar density profile is studied on the simple cubic lattice for  $l = 20, 40, 60, 80$  and 100 (see figure 7).

Recently Coniglio and Zannetti [9] proposed a multiscaling form for the density profile  $g(r, R)$  for the growth of any fractal object of average size  $R$ . From group theoretical arguments they found that the density profile should follow a multiscaling form

$$g(r/R) = r^{-d+D(r/R)} A(r/R). \quad (5)$$

This multiscaling form says that every region within the fractal object with a fixed value of  $x = r/R$  has a distinct fractal dimension  $D(r/R)$  rather than a constant fractal dimension  $D$  for the cluster as a whole. In fact Coniglio and Zannetti note that scale invariance cannot predict whether a single exponent ( $D(r/R) = \text{a constant}$ ) or many exponents are present in the growing cluster. This may only be determined from experimental data or further theoretical calculations. Following this idea we calculated  $g(x)$  for a fixed value of  $x$  but for different chemical distances corresponding to different  $r$  and  $R$  values. We plot them in a double logarithmic scale for all five chemical distances and the slope of this straight line gives us the value of  $D(x) - d$ . To get the

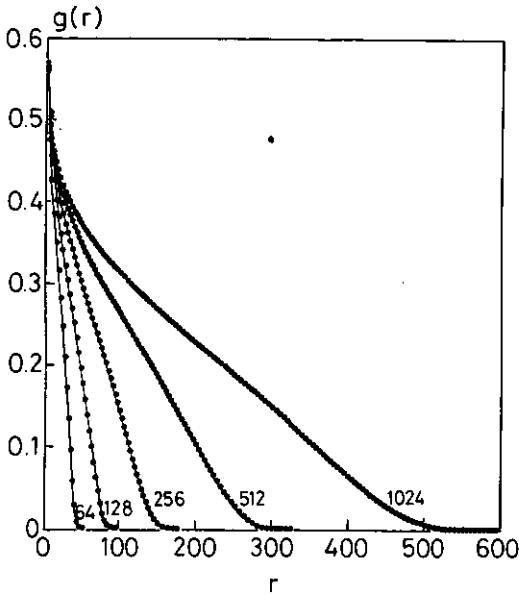


Figure 6. Density profiles  $g(r, R)$  for chemical distances  $l = 64, 128, 256, 512$  and  $1024$  on the square lattice.

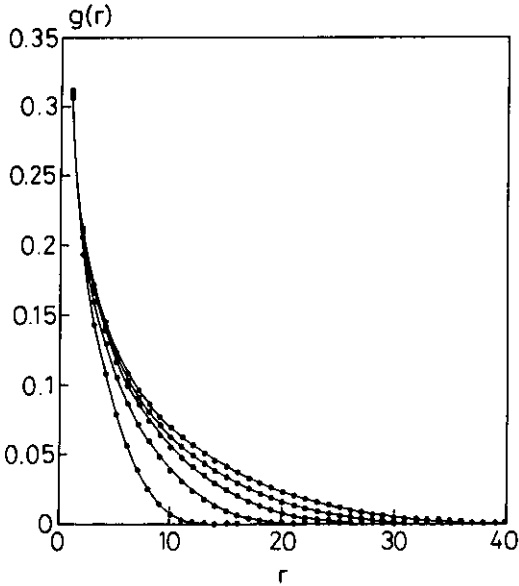


Figure 7. Density profiles  $g(r, R)$  for chemical distances  $l = 20, 40, 60, 80$  and  $100$  on the simple cubic lattice.



fractal dimension at the central region ( $x \rightarrow 0$ ) one needs big clusters to reduce the error. We consider only two values of the chemical distances  $l = 512$  and  $l = 1024$  and estimate  $D(x) - d$  by calculating  $\log(g_1(x)/g_2(x))/\log(r_1/r_2)$ . We varied the value of  $x$  between  $x = 0.05$  and  $1.5$  at intervals of  $0.05$  and corresponding values of  $D(x) - d$  are plotted with  $x$  (see figure 8(a)). We fit a straight line through the first five points and extrapolation to  $x \rightarrow 0$  gives the value of  $D(0) - d = -0.105$ . The fractal dimensionality of the percolating cluster is identical to the magnetic exponent  $y_h$  of the  $q$ -state Potts model in the limit  $q \rightarrow 1$  (Kapitulnik *et al* [10] and references therein)

$$d_f = y_h = d - \beta/\nu.$$

In two dimensions Nienhuis [11] showed via RG mappings that  $y_h$  is  $91/48 = 1.89583 \dots$  which is in good agreement with our numerical value of  $1.895$ .

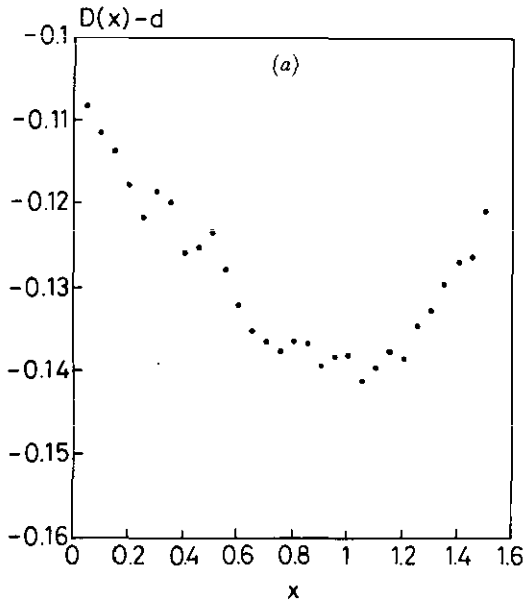


Figure 8(a).  $D(x) - d$  is plotted against  $x$  for the square lattice. The extrapolated value as  $x = 0$  is  $0.105$ .

In three dimensions we get a strongly fluctuating curve (see figure 8(b)) where we have used data for the chemical distances  $l = 80$  and  $100$  to get  $D(x) - d$ . If one tries to pass a straight line through this fluctuating data, say from the  $x = 0.05$  to  $0.40$  or from  $0.05$  to  $0.65$ , and extrapolate them to  $x \rightarrow 0$  one gets  $D(0) - d = -0.499$  and  $-0.511$ . Thus the fractal dimensionality of the percolating cluster in three dimensions is  $2.50 \pm 0.01$  which is in good agreement with other numerical reports. With this analysis we would like to stress that the multiscaling distribution (5) is a good method of calculating the fractal dimension of a cluster. About multiscaling, we see that our results (figures 8(a) and 8(b)) show very little evidence of such scaling in the percolation growth process.

Finally we would like to discuss the possible multifractality in this growth process. If a growth process is characterized by a probability distribution, whose different moments scale with different exponents and these exponents vary in a nonlinear fashion, the system is said to possess multifractal behaviour. Here we study two probability

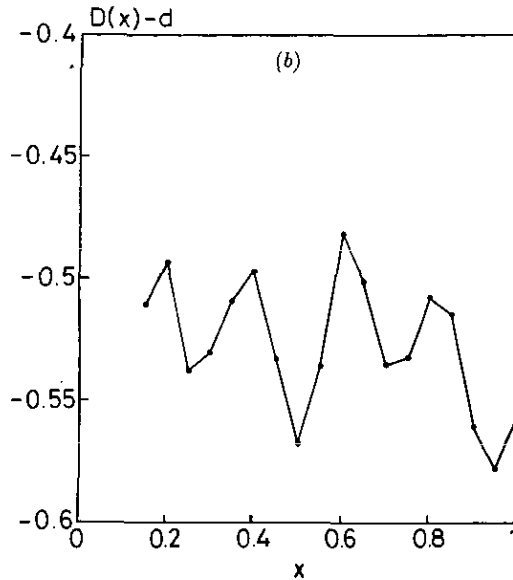


Figure 8(b).  $D(x) - d$  is plotted against  $x$  for the simple cubic lattice. The extrapolated value as  $x=0$  is 0.50.

distributions,  $P(r, R)$  and  $g(r, R)$ , and we are unable to detect multifractality. Exponents characterizing the different moments vary in a linear manner, indicative of normal 'gap' scaling.

In conclusion we have studied the growth probability distribution for the single cluster growth process for percolation at the percolation threshold for both two and three dimensions. For two dimensions, we found this distribution indistinguishable from the distribution function obtained by Havlin *et al* [7] for the chemical path distribution on the percolation cluster. We have presented evidence that the average radius of the growing zone and the width of the interface scale with the same exponent  $\nu$ . We have also shown that the multiscaling form by Coniglio and Zannetti [9] can be used to get the fractal dimension accurately. We do not find any evidence of multiscaling or multifractality in this growth process.

One of us (SSM) gratefully acknowledges financial support through a St Francis Xavier University post-dostoral fellowship. NJ is supported by a NSERC (Canada) grant.

## References

- [1] Plischke M and Racz Z 1984 *Phys. Rev. Lett.* **53** 415
- [2] Meakin P, Coniglio A, Stanley H E and Witter T 1986 *Phys. Rev. A* **34** 3325
- [3] Leath P L 1976 *Phys. Rev. B* **14** 5046
- [4] Alexandrowicz Z 1980 *Phys. Lett.* **80A** 284
- [5] Ziff R M and Sapoval B 1986 *J. Phys. A: Math. Gen.* **19** L1169
- [6] Ziff R M and Stell G (unpublished)
- [7] Havlin S, Trus B, Weiss G H and Ben-Avraham D 1985 *J. Phys. A: Math. Gen.* **18** L247
- [8] de Gennes P G 1979 *Scaling Concepts in Polymer Physics* (Ithaca, NY: Cornell University)
- [9] Coniglio A and Zannetti M 1989 *Physica D* **38** 37
- [10] Kapitulnik A, Aharony A, Deutscher G and Stauffer D 1983 *J. Phys. A: Math. Gen.* **18** L269
- [11] Nienhuis B 1984 *J. Stat. Phys.* **34** 731